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A Note on the Nuclear Excited Feshbach Resonance

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The scattering amplitude and the wave function for the final bound state of an electron in Turner's [1] mechanism are modified. The life time for the temporarily formed negative ion and the capture cross section in the case of electron-polar molecule interaction are reported.

It is observed that the effect of our modification is to increase the life time and to decrease the capture cross section. Our results favour the existence of long lived parent negative ions.

In spite of the large amount of work done on the subject of the scattering of slow electrons by polar molecules there still exists a controversy regarding the large diffusion cross section of some of the polar molecules. In order to explain the large scattering cross section Turner [1] proposed that there exists a momentary capture of the slow electron with a rotational excitation of the polar molecule. It is well known that, though this was a major breakthrough, some facts about the large capture time remain unexplained. Recently, Christophorou [2–4] has reported that the life time of negative ions in the captured state is much larger than predicted theoretically. Calculations were also made by Itikawa [5] for the cross section of electron scattering by polar molecules, taking into account the capture and decay processes in unified manner. The large cross section may also be attributed to short range forces [6, 7]. However, recent experiments [2–4] favour the Feshbach type resonance as predicted by Turner.

In this short communication we have made an attempt to find out the consequences of using simultaneously a better approximation for the scattering amplitude and a slightly different wave function for the bound state of the electron in the dipole field. Our corrections favour the existence of long lived negative parent ions.

For the determination of the scattering amplitude the variational method as established by Mower [8] on the basis of the variational method developed by Schwinger [9], is used. The expression for the trial wave functon needed in this method is of the form $Ce^{i\,ks}$. By the use of this trial function the scattering amplitude is obtained as

$$f(\theta) = f_{\mathrm{B}}(\theta)\,[1 + T_0(\theta)]\,. \tag{1}$$

$f_{\mathrm{B}}(\theta) = 2\,i\,D\,e\,m\,\nu'/\hbar^2\,k$ is the amplitude using the Born approximation and $T_0(\theta) = (i\,k/2)\,f_{\mathrm{B}}(\theta)$, where m is the mass of the electron. ν' is the direction cosine of the dipole with respect to the wave vector $\boldsymbol{K}$. The magnitude of $\boldsymbol{K}$ is given by $K_0[2(1-\cos\theta)]^{1/2}$. θ is the angle between the initial and the scattering direction. It is observed that the use of the trial function gives an amplitude factor slightly larger than the Born approximation scattering amplitude $f_{\mathrm{B}}(\theta)$. The diffusion cross section also increases.

For the final bound state we choose a wave function of the electron which is the sum of the S and P states as given by [1] but differing from that in the radial part. Thus, we write a trial function for the final bound state of the electron

$$\psi_{\mathrm{f}} = \frac{N}{1 + p^2\,\zeta^2}\,\{C_0\,Y_{00}(\theta, \Phi) + C_1\,Y_{10}(\bar{\theta}, \Phi)\} \tag{2}$$

where N is the normalization constant and $\bar{\theta}$, $\bar{\Phi}$ are the angles of the electron relative to the dipole axes. p is a variable parameter. The normalization was carried out in the usual manner requiring that $|C_0|^2 + |C_1|^2 = 1$.

The matrix element T_{f_i} for the transition probability was calculated in the same manner as Turner. Simultaneous application of both corrections leads to the expression

$$\begin{aligned} T_{\mathrm{f}_i} = \frac{i\,D\,e\,m}{I\,K\sqrt{6\,p\,\pi}} & \left\{ (5\,\pi)^{1/2} \left[S_{\mathrm{f}_i}{}^{(1)} - \frac{2\,D\,e\,m}{3\,\hbar^2}\,\Delta S_{\mathrm{f}_i}{}^{(1)} \right] \right. \\ & \left. + \frac{(3\,\pi)^{1/2}}{2} \left[S_{\mathrm{f}_i}{}^{(3)} - \frac{4\,D\,e\,m}{3\,\hbar^2}\,\Delta S_{\mathrm{f}_i}{}^{(3)} \right] \right\} \left\{ e^t\,E_i(-t) \right. \\ & \left. + e^{-t}\,E_i(t) + \frac{i\,\pi}{2}\,e^{-t} \right\}. \end{aligned} \tag{3}$$

Table 1.

Quantity	Turner's results	Values due to amplitude correction only	Values due to wave fn. correction only	Values due to simultaneous correction of both factors
Matrix element of the transition probability $\lvert T_{\mathrm{f}_i}\rvert^2 \times 10^{-48}$	3.6	2.88	1.95	1.56
Life time of negative ion in sec $\times 10^{-13}$	1.3	1.72	3.024	3.78
Capture cross section in cm^2 $\times 10^{-14}$	2.4	1.92	1.299	1.0392

In expression (3) the corrections due to the variational approach to the scattering amplitude are

$$-\frac{2\,D\,e\,m}{3\,\hbar^2}\,\Delta S_{\mathrm{f}_i}{}^{(1)} \quad \text{and} \quad -\frac{4\,D\,e\,m}{3\,\hbar^2}\,\Delta S_{\mathrm{f}_i}{}^{(3)}$$

while the modification due to the use of Eq. (2) is

$$\left\{ e^t\,E_i(-t) + e^{-t}\,E_i(t) + \frac{i\,\pi}{2}\,e^{-t} \right\}$$

where t is equal to K/P. $S_{\mathrm{f}_i}{}^{(1)}$; $S_{\mathrm{f}_i}{}^{(3)}$; $\Delta S_{\mathrm{f}_i}{}^{(1)}$; $\Delta S_{\mathrm{f}_i}{}^{(3)}$ are defined in the appendix.

Considering an experiment at room temperature, with the help of Eq. (3) the calculations of the life time of a temporarily formed negative ion and the capture cross section are made exactly as Turner. The dipole moment D was taken to be 2×10^{-18} esu. cm. and I of the order of 10^{-40} gm. cm^2. In Table 1 results for the life time and capture cross section for the $L=2$ and $L=3$ states are exhibited.

It is observed that the correction term depends on the dipole moment and also on the velocity of the electron. The modification in the calculation is to increase the life time nearly three times and to decrease the capture cross section about 2.3 times.

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Appendix

The terms $S_{\mathrm{f}_i}{}^{(1)}$; $S_{\mathrm{f}_i}{}^{(3)}$; $\Delta S_{\mathrm{f}_i}{}^{(1)}$; $\Delta S_{\mathrm{f}_i}{}^{(3)}$ used in Eq. (3) are as follows.

$$S_{\mathrm{f}_i}{}^{(1)} = L_\mathrm{f}(L_\mathrm{f}+1)\,[\alpha(L_i\,M_i)\,\delta_{L_\mathrm{f}+1,\,L_i+1} + \beta(L_i\,M_i)\,\delta_{L_\mathrm{f},\,L_i-1}]\,\delta_{M_\mathrm{f} M_i}$$

$$\Delta S_{\mathrm{f}_i}{}^{(1)} = L_\mathrm{f}(L_\mathrm{f}+1)\,\delta_{L_\mathrm{f} L_i}\,\delta_{M_\mathrm{f} M_i}\;.$$

$$\begin{aligned} S_{\mathrm{f}_i}{}^{(3)} = \{ & L_i(L_i-1)\,[\gamma(L_\mathrm{f}\,M_\mathrm{f})\;\eta(L_i\,M_i) + \in(L_\mathrm{f}\,M_\mathrm{f})\,\xi(L_i\,M_i)] \times \delta_{L_\mathrm{f}+1,\,L_i-1} \\ & + (L_i+1)\,(L_i+2)\;[\gamma(L_i\,M_i)\,\eta(L_\mathrm{f}\,M_\mathrm{f}) + \in(L_i\,M_i)\,\xi(L_\mathrm{f}\,M_\mathrm{f})]\,\delta_{L_\mathrm{f}-1,\,L_i+1}\,\}\,\delta_{M_\mathrm{f} M_i} \end{aligned}$$

$$\begin{aligned} \Delta S_{\mathrm{f}_i}{}^{(3)} = {} & \tfrac{1}{3}\,[L_i(L_i+1)-2]\,\delta_{L_\mathrm{f} L_i}\,\delta_{M_\mathrm{f} M_i} - \tfrac{1}{5}\,[L_i(L_i+1)-4+i] \\ & \{\alpha(L_i\,M_i)\,\delta_{L_\mathrm{f} L_i+1} + \beta(L_i\,M_i)\,\delta_{L_\mathrm{f},\,L_i-1}\}\,\delta_{M_\mathrm{f} M_i} + \tfrac{2}{5}\,\{L_i(L_i+1) \\ & + (M_i-2)\}\{\gamma(L_i\,M_i) + \eta(L_i\,M_i)\}[\in(L_\mathrm{f}\,M_\mathrm{f})\,\sigma(L_i\,M_i)\,\delta_{L_\mathrm{f}+1,\,L_i+2} \\ & - \in(L_\mathrm{f}\,M_\mathrm{f})\,\varrho(L_i\,M_i)\,\delta_{L_\mathrm{f}+1,\,L_i} + \xi\,(L_\mathrm{f}\,M_\mathrm{f})\,\sigma(L_i\,M_i)\,\delta_{L_\mathrm{f}-1,\,L_i+2} + \xi(L_\mathrm{f}\,M_\mathrm{f})\,\varrho(L_i\,M_i)\,\delta_{L_\mathrm{f}-1,\,L_i}\,]\,\delta_{M_\mathrm{f}+1,\,M_i-1} \end{aligned}$$

were

$$\alpha(L,M) = \beta(L+1,M) = \left[\frac{(L+M+1)\,(L-M+1)}{(2\,L+1)\,(2\,L+3)}\right]^{1/2}$$

$$\gamma(L,M) = \eta(L+1,\,-M+1) = \left[\frac{(\mathrm{L}-M+1)\,(L-M+2)}{(2\,L+1)\,(2\,L+3)}\right]^{1/2}$$

$$\varrho(L,M) = = \sigma(L+1,M) = \left[\frac{(L+M)\,(L-M+2)}{(2\,L+1)\,(2\,L+3)}\right]^{1/2}$$

$$\in(L,M) = \xi(L+1,\,-M-1) = \left[\frac{(L+M+1)\,(L+M+2)}{(2\,L+1)\,(2\,L+3)}\right]^{1/2}.$$

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